



Generation of Sub Wavelength Focal Hole Segment Using Azimuthally Polarized Annular Multi Gaussian Beam by High NA Parabolic Mirror

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Abstract

We investigate the focusing properties of azimuthally polarized annular multi Gaussian beam with a high numerical aperture parabolic mirror are numerically investigated by vector diffraction theory. We observe that our proposed system generates a sub wavelength focal hole of 0.35λ having large uniform focal depth of 10λ . This kind of nondiffracting focal hole is called dark channel, which may have possible applications in particle acceleration, optical trapping and manipulating, single molecule imaging and high resolution imaging microscopy.

Keywords: Azimuthally polarized beam; High NA Mirror; Vector diffraction theory; Optical Trapping.

1. INTRODUCTION

In recent years, research interest in the tight focusing of light beams with azimuthally polarized beams with long depths of focus (DOFs) have been steadily growing for their practical applications in high-density optical data storage, photolithography, high-resolution imaging, and other uses. Optical bottle beams are beams with low or null intensity channels surrounded by three-dimensional (3D) regions of higher intensity. Over the past few years, a variety of techniques have been proposed for generating such beams for applications in optical tweezers and atom traps (Arlt and Padgett 2000; Ahluwalia *et al.* 2004, Rudy *et al.* 2001, Yelin *et al.* 2004, Pu *et al.* 2005, Isenhower *et al.* 2009, Xu *et al.* 2010, Veerabagu *et al.* 2013). Stable trapping of a single particle is expected if we can make the bottles small enough and comparable to the particle size. Such “microbottles” were established recently with the volume speckle field (Shvedov *et al.* 2010). Recently, a sub wavelength focal hole (0.5λ) with a quite long depth of focus (26λ) was achieved near the focus by tight focusing of a double-ring-shaped azimuthally polarized beam with annular high numerical aperture (NA) lens (Tian *et al.* 2011, Lalithambigai *et al.* 2012, Prabakaran *et al.* 2013). Diffractive optical elements have been used to achieve

long DOFs, and they are able to reduce the compromise between the NA and the DOF that determine the lateral and axial resolutions in a conventional optical imaging system, respectively (Golub *et al.* 2006). An achromatic hybrid refractive-diffractive lens with an extended DOF that works in the entire visible light band was designed and fabricated (Flores *et al.* 2004). An extremely narrow annular aperture was proven as an efficient method for producing a non diffracting beam but with dramatically reduced intensity (Durnin 1987). A non diffracting beam can also be realized by placing multi belt binary optical phase elements (Wang *et al.* 2008) or an amplitude filtering mask (Lin *et al.* 2011) on the lens pupil. Though the proposed method is simple and free from the complex interferometric methods used in the conventional techniques to generate focal hole, it should be noted that a lot of energy of the incident light beam is blocked by the central part of the annular aperture. In another study, a sub-wavelength light needle with a longer depth of focus has been generated using dual beam focusing (Kuang *et al.* 2011). This dark channel may have many applications, ranging from atom optics to single molecule detection, in which the dark regions of zero intensity are required (Klar *et al.* 2001, Engel *et al.* 2003, Helseth 2001). Recently, radially polarized annular multi-Gaussian beam mode is proposed for

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illuminating the pupil plane of the objective to achieve sub wavelength longitudinal beam with long focal depth (Jian-Nong *et al.* 2011). Specifically, for the aplanatic lens, $A(\theta) = \cos^{1/2} \theta$; and for the parabolic mirror, $A(\theta) = 2/(1+\cos\theta)$. It means that, due to energy conservation, in a parabolic mirror much of the incident energy reaches the focus under high angles while the contrary is valid for an objective lens. In this Letter, we investigate the focused properties of a azimuthally polarized annular multi Gaussian beam by a high NA parabolic mirror. We find that the depth of focus of the sub wavelength size. It is observed that the proposed method is simple and free from the complex interferometric methods used in the conventional techniques to generate focal hole.

2. THEORY

At optical frequencies, only the electric field is responsible for the interaction with matter (scattering, fluorescence, excitation, polarization, etc.). Hence only the electric fields will be discussed. Instead of a azimuthal polarization pattern as shown in Fig. 1(a). Such a beam is incident upon a high aperture parabolic mirror and then focused on the focus as shown in Fig. 1 (b).

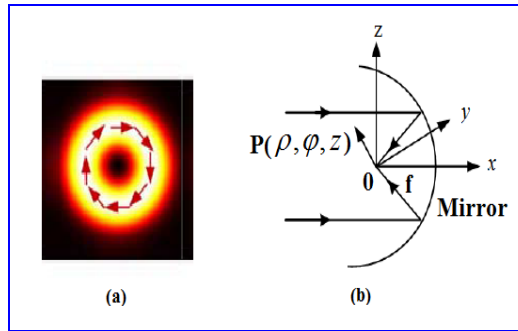


Fig. 1: (a) Schematic of azimuthally polarized annular multi Gaussian beam. (b) Diagram of the focusing configuration

The analysis was performed on the basis of Richards and Wolf's vectorial diffraction method widely used for high-NA mirror system at arbitrary incident polarization. Mathematically, the generalized azimuthally polarized beam can be expressed as.

$$E(\rho, \varphi) = l_0(r)(\cos \varphi_0 e_\rho + \sin \varphi_0 e_\varphi) \exp(in\varphi) \rightarrow (1)$$

where $l_0(r)$ is complex amplitude distribution of the beam, $\exp(in\varphi)$ is a helical phase term and n the topological charge. The Richards–Wolf vectorial

diffraction theory, the electric field near the focus at point P can be obtained

$$E^s = -\frac{ikf}{2\pi} \int_{\alpha_0}^{\alpha} \int_0^{2\pi} E_f(\theta, \varphi) e^{ik_s \cdot r_p} \sin \theta d\theta d\varphi \rightarrow (2)$$

Where E_f is the electric field at the focal sphere, which can be calculated by geometrical optics rules (Zhan *et al.* 2002)

$$E_f = \frac{2l_0(\theta)}{1+\cos\theta} \cos\theta \cos\varphi e_x + \cos\theta \sin\varphi e_y + \sin\theta e_z \rightarrow (3)$$

where $2/(1+\cos\theta)$ and $l_0(\theta)$ are the apodization factor and pupil apodization function for the parabolic mirror, respectively.

$$e^{ik_s \cdot r_p} = \exp[(-ik_z \cos\theta) - ik_\rho \sin\theta \cos(\varphi - \varphi_s)] \rightarrow (4)$$

where k is wave number and f the focal length. Substituting Eq. (3) and Eq. (4) into Eq. (2) and transforming cartesian coordinates to cylindrical coordinates, the azimuthally components of E can be calculated as follows

$$E_\varphi^s(\rho_s, \varphi_s, z_s) = \frac{(-i)^n kf}{2} \exp(in\varphi) \sin \varphi_0 \int_{\alpha_0}^{\theta_{\max}} \frac{2 \sin \theta l_0(\theta)}{1+\cos\theta} A(\theta) [J_{n+1}(k\rho \sin\theta) - J_{n-1}(k\rho \sin\theta)] \exp(-ikz_s \cos\theta) d\theta \rightarrow (5)$$

$A(\theta)$ describes the amplitude-modulated annular multi-Gaussian beam, this function is given by (Nong *et al.* 2011)

$$A(\theta) = \left(\frac{\theta}{\theta_0}\right)^m \sum_{n=-N}^N \exp\left[-\left(\frac{\theta - \theta_c - n\omega_0}{\omega_0}\right)^2\right] \rightarrow (6)$$

Here, θ is the converging semi-angle. We denote the maximum converging semi-angle as θ_{\max} which is related to objective numerical aperture by $\theta_{\max} = \arcsin(\text{NA})$, along with integer m determines the shape of the modulation function.. Here we take $\theta_c = \theta_{\max}/2$. w_0 is the waist width of single Gaussian beam which is calculated by the following formula

$$\omega_0 = 1/2 \times \frac{\theta_{\max}}{N + \left\{1 - \ln \left[\sum_{n=-N}^N \exp(-n^2) \right]\right\}^{1/2}} \rightarrow (7)$$

3. RESULTS

We perform the integration of Eq.(5) numerically for $\text{NA}=0.9$ and $\lambda=1$, $\theta_{\max}=64^\circ$, $\theta_0=62^\circ$ and $\theta_c=\theta_{\max}/2$, $w_0=0.0364$, $N=20$ and $m=20$. For all calculation in the length unit is normalized to λ and the energy density is normalized to unity. Fig.(2) illustrates

the evolution of three-dimensional light intensity distribution of high NA mirror for the incident plane azimuthally polarized beam which is shown in Fig.(2.a & b). In this case, only the azimuthal component is present at the focus and the focal segment resembles a doughnut shape. From the fig(2.b) It is observed that the FWHM of the generated focal hole is 0.4λ . The 3D intensity distribution near the focus as shown in Fig.(2.a), reveals that the focal depth of the generated focal hole segment is only 1λ .

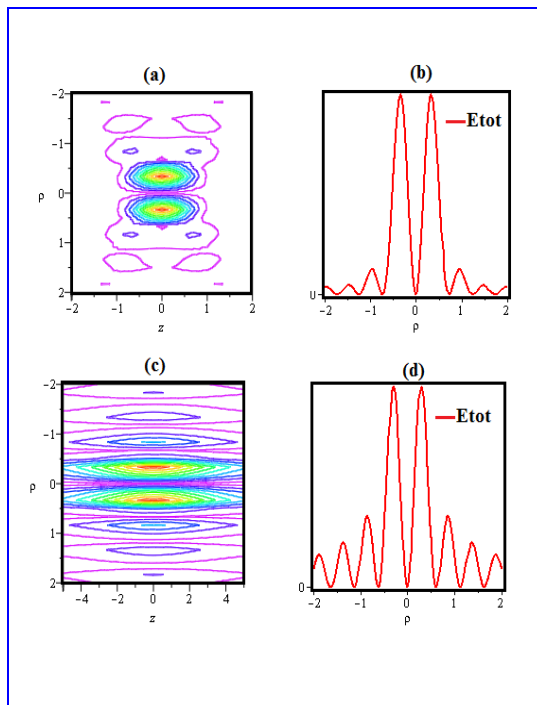


Fig. 2: (a) Contour plot for the total intensity distribution in the p-z plane for azimuthally polarized parabolic mirror. (b) 2D Intensity distributions at the focus of the mirror at $z=0$. (b) & (d) are same as (c) and (d) but for azimuthally polarized annular multi gaussian beam

We also observed that it is possible to generated sub wave length focal hole segment by properly determines the shape of the modulation function of incident annular multi Gaussian beam. However for the annular multi gaussian beam ,we measured the FWHM of the generated focal hole as 0.35λ and its focal depth improved as 10λ which are shown in fig.2(d& c) respectively. Such a sub wavelength focal hole segment is highly useful for trapping multiple particles with a dielectric constant lower than the ambient.

4. CONCLUSION

In conclusion, based on the vector diffraction theory, the focusing property of high NA parabolic mirror for the incident azimuthally polarized annular multi Gaussian beams is studied numerically using vector diffraction theory. It is observed that highly confined sub wavelength focal hole with large focal depth. So the high NA parabolic mirror can be an especially useful alternative for focusing and beam-shaping. Such a focal hole segment is an effective tool for optical trapping for low refractive index particles can be achieved precisely and controllably.

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